Two and a Half Perspectives on Semiclassical Strings in $AdS_4 \times CP^3$

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Work (and work in progress) with

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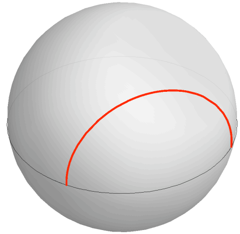
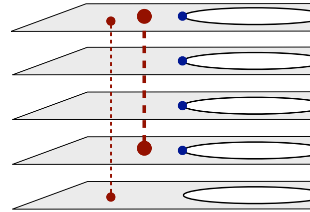
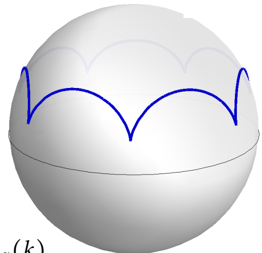
Topics:

1 Energy corrections for giant magnons in algebraic curve

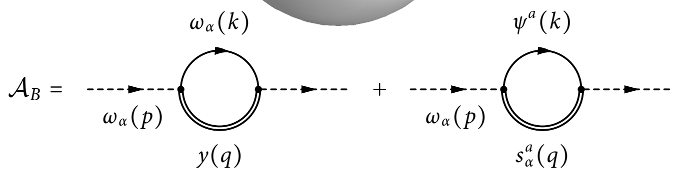


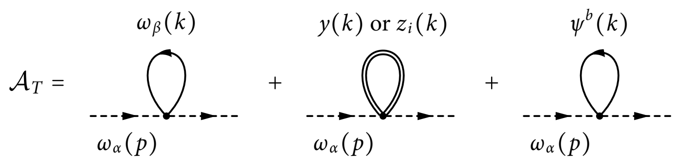


1½ ...and for finite-J giant magnons



2 Near-flat-space (and BMN) mass corrections





Topics:

1 Energy corrections for giant magnons in algebraic curve

1½ ...and for finite-*J* giant magnons

2 Near-flat-space (and BMN) mass corrections

Heavy modes…

Divergences involved:

$\log \Lambda_{\text{heavy}} - \log 2\Lambda_{\text{light}}$

are composite in off-shell construction

and Λ^2 in finite-J case.

$\log \Lambda$ and Λ^2 diagrams

(both $1/\varepsilon$ in dim. reg.)

can decay at one loop

New example of AdS/CFT: [Aharony, Bergman, Jafferis, Maldacena, 2008]

$$\mathcal{N} = 6 \text{ CSM theory (in 2+1 dimensions)} \quad \longleftrightarrow \quad \text{IIA strings in } AdS_4 \times CP^3$$

Scalar fields are in $(N, \bar{N})$ of $U(N) \times U(N)$ (rather than adjoint), and the spin chain vacuum is

$$O = \mathrm{Tr}\left(Y_1 Y_4^\dagger\right)^J$$

Can excite even or odd chain, decoupled at leading order: [Minahan & Zarembo, 2008]

$$\Delta - \frac{J}{2} = \sum_{i/2} (\mathcal{H}_{i,i+2} + \mathcal{H}_{i+1,i+3}) + \text{ four loops}$$

Remaining symmetries fix the exact dispersion relation

$$E = \Delta - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 4\, h(\lambda)^2 \sin^2 \frac{p}{2}}$$

but leave the function $h(\lambda)$ unknown.
(This was true in $AdS_5 \times S^5$ case too, but there $f(\lambda) = \lambda$ exactly.)

Weak coupling:

$$h(\lambda)^2 = \lambda^2 - 4\,\zeta(2)\,\lambda^4 + \ldots \qquad \lambda = \frac{N}{k} \ll 1$$

[Leoni, Mauri, Minahan, Ohlsson Sax, Santambrogio, Sieg, Tartaglino-Mazzucchelli 2010]

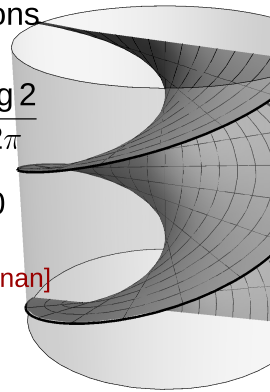
Strong coupling:

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + c + \frac{d}{\sqrt{\lambda}} + \ldots \qquad \lambda = \frac{R^4}{32\pi^2\alpha'^2} \gg 1$$

where from spinning strings:

$$\frac{\Delta - S}{\log S} = 2h(\lambda) - 3\frac{\log 2}{2\pi} + o\left(\frac{1}{h}\right) \qquad \text{from } sl(2) \text{ Bethe equations}$$

$$= \sqrt{2\lambda} + \begin{cases} -5\dfrac{\log 2}{2\pi} & \\ & \text{using} \\ -3\dfrac{\log 2}{2\pi} & \end{cases} \begin{matrix} \text{old sum} \\ \\ \text{new sum} \end{matrix} \quad \Rightarrow \quad \begin{matrix} c_{\text{old}} = -\dfrac{\log 2}{2\pi} \\ \\ c_{\text{new}} = 0 \end{matrix}$$



[Gromov & Vieira] [McLoughlin & Roiban] [Alday, Arutyunov, Bykov] [Krishnan]
[McLoughlin, Roiban, Tseytlin] [Gromov & Mikhaylov] 2008

String energy corrections are

$$\delta E = \sum_{n,k} (-1)^{F_k} \frac{\hbar}{2} \omega_n^k$$

Modes are of course perturbations like this

$$X^\mu_{\text{classical}} + e^{-i\omega_n t} \delta X^\mu_n$$

$$\to \text{plane waves } e^{ikx - i\omega t \pm i\delta/2} \text{ as } x \to \pm\infty, \text{ with } \begin{cases} \omega^2 = k^2 + 1 & \text{heavy} \\ \omega^2 = k^2 + 1/4 & \text{light} \end{cases}$$

($\exists$ subspaces radius R and $R/2$)

Cutoff prescriptions:

$$\delta E_{\text{old}} = \lim_{N\to\infty} \sum_{n=-N}^{N} \left(\omega_n^{\text{light}} + \omega_n^{\text{heavy}} \right)$$

$$\delta E_{\text{new}} = \lim_{N\to\infty} \left(\sum_{n=-N}^{N} \omega_n^{\text{light}} + \sum_{n=-2N}^{2N} \omega_n^{\text{heavy}} \right)$$

Heavy or light alone diverge as $\log N$. [Gromov & Mikhaylov, 2008]

Giant Magnons

These are classical string solutions having exactly the fundamental dispersion relation:

$$\Delta - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 4\, h(\lambda)^2 \sin^2 \frac{p}{2}}$$

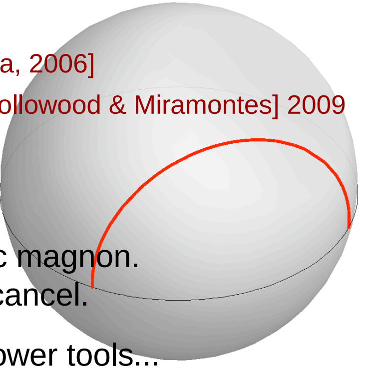
$$= \sqrt{\frac{Q^2}{4} + 2\lambda \, \sin^2 \frac{p}{2}} + \frac{c\,\sqrt{8\lambda}\sin^2 \frac{p}{2}}{\sqrt{\frac{Q^2}{4} + 2\lambda\, \sin^2 \frac{p}{2}}} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right)$$

In $AdS_4 \times CP^3$ we have:

$Q = 1$	$CP^1 = S^2$	same as [Hofman & Maldacena, 2006]
$Q \sim \sqrt{\lambda}$	CP^2 dyonic	[MCA, Aniceto, Ohlsson Sax] [Hollowood & Miramontes] 2009

and superimposing two of these,

$Q = 1$	$RP^2 \approx S^2$	
$Q \sim \sqrt{\lambda}$	$RP^3 \approx S^3$	same as [Dorey, 2006]'s dyonic magnon.
$Q = 0$	"big"	a solution in which $+Q - Q$ cancel.



Finding modes $\delta X^\mu(x, t)$ by hand is hard, so we use power tools...

Algebraic Curve:

Classical string solutions $\underset{\text{one-to-one}}{\longleftrightarrow}$ Riemann surfaces

Construction from Lax connection:

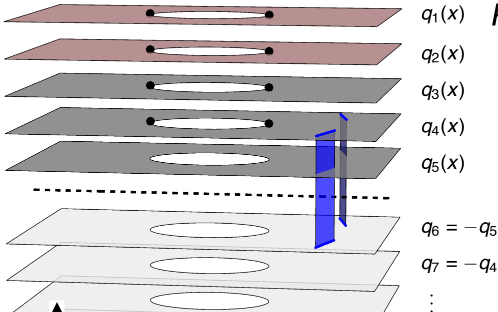
$$M(x) = P\exp\oint d\sigma\, J_\sigma(x) \qquad \mathrm{eig}M = \left\{e^{ip_1(x)}, e^{ip_2(x)}, e^{ip_3(x)}, \ldots\right\}$$

Giant magnons have a single log cut:

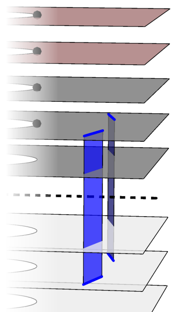
$$q_4(x) = \alpha\frac{x}{x^2-1} - i\,\log\left(\frac{x-X^+}{x-X^-}\right) - \frac{p}{2}$$

Choice of sheets determines type...

Elementary:



RP^3:



Well-developed scheme for **semiclassical perturbations**:

- Add $\sqrt{}$ cut connecting sheets (i,j)
- at point y solving $q_i(y) - q_j(y) = 2\pi n$
- with filling fraction $S_{ij} = \frac{g}{i\pi}\oint_{C_{ij}} dx \left(1 - \frac{1}{x^2}\right) q_i(x) = 1$

[Beisert, Kazakov, Sakai, Zarembo, 2005]
[Gromov, Vieira, 2007]

Light modes connect to sheet 5 or 6,
heavy modes do not.

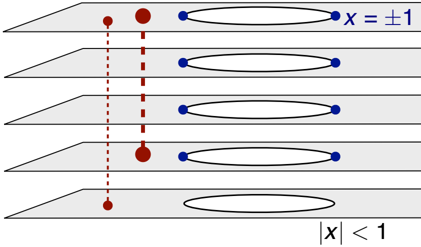
After constructing mode $\delta q_i(x)$, read off its perturbation of the energy:

$$\delta\Delta = \Omega_{ij}(y) = \omega_n^{ij}$$

For giant magnons, the "the off-shell frequencies" are:

$$\Omega_{\text{light}}(y) = \frac{1}{y^2 - 1}\left(1 - y\,\frac{X^+ + X^-}{1 + X^+X^-}\right)$$

$$\Omega_{\text{heavy}}(y) = \frac{2}{y^2 - 1}\left(1 - y\,\frac{X^+ + X^-}{1 + X^+X^-}\right)$$



Only change from S^5 is the factor of 2.

Not easy to find positions x_n^{ij}, hence “on-shell” frequencies $\omega_n^{ij} = \Omega_{ij}(x_n^{ij})$.

Can still add them up, with an integral: [Schäfer-Nameki 2006]

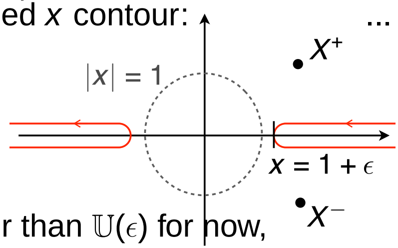
$$\delta E = \frac{1}{2}\sum_n^N (-1)^{F_{ij}}\,\Omega_{ij}(x_n^{ij})$$

$$2\pi n = q_i(x) - q_j(x)$$

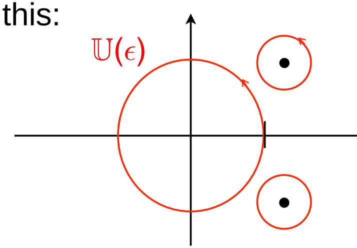
$$= \frac{1}{4i}\sum_{ij}(-1)^{F_{ij}} \oint_{\mathbb{R}(N)} dn\;\cot(\pi n)\Omega_{ij}(x_n^{ij})$$

$$= \frac{1}{4i}\sum_{ij}(-1)^{F_{ij}} \oint_{-\mathbb{U}(\epsilon)} dx\,\frac{q_i'(x) - q_j'(x)}{2\pi}\cot\left(\frac{q_i(x) - q_j(x)}{2}\right)\Omega_{ij}(x)$$

where we have unwrapped x contour:



... to this:



Ignore components other than $\mathbb{U}(\epsilon)$ for now, but must worry about cutoffs.

New sum is simplest: $x^{\text{heavy}}_{2N} \approx x^{\text{light}}_{N}$, thus cut off at same $x = 1 + \epsilon$ for both:

$$\delta E_{\text{new}} = \lim_{\epsilon\to 0} \sum_{ij} \oint_{\mathbb{U}(\epsilon)} dx \; \frac{(-1)^{F_{ij}}}{-4i} \frac{q_i' - q_j'}{2\pi} \cot\left(\frac{q_i - q_j}{2}\right) \Omega_{ij}(x)$$
$$= 0$$

(Done by [Shenderovich, 2008], before [Gromov & Mikhaylov, 2008] wrote the sum...)

Old sum is more work, $x^{\text{heavy}}_{N} - 1 \approx 2(x^{\text{light}}_{N} - 1)$ so

$$\delta E_{\text{old}} = \lim_{\epsilon\to 0} \left\{ \sum_{ij\text{ light}} \oint_{\mathbb{U}(\epsilon)} dx + \sum_{ij\text{ heavy}} \oint_{\mathbb{U}(2\epsilon)} dx \right\} \frac{(-1)^{F_{ij}}}{-4i} \frac{q_i' - q_j'}{2\pi} \cot\left(\frac{q_i - q_j}{2}\right) \Omega_{ij}(x)$$
$$= \frac{-\log 2}{2\pi} 2 \sin \frac{p}{2}$$

Dyonic case: $\delta E_{\text{old}} = \frac{-\log 2}{2\pi} \frac{\sqrt{8\lambda}\sin^2\frac{p}{2}}{\sqrt{\frac{Q^2}{4} + 2\lambda\sin^2\frac{p}{2}}}$. [MCA, Aniceto, Bombardelli, 2010]

Again heavy and light are each $\log\epsilon$ divergent;
this is where $c = -\frac{\log 2}{2\pi}$ comes from.

Suppose you wanted a **physical cutoff**: always the same energy Λ:

$$\delta E_{\text{phys}} = \frac{1}{2}\sum_{ij}\sum_{n=-N_{ij}}^{N_{ij}} (-1)^{F_{ij}}\omega_n^{ij} \qquad\qquad \omega_{N_{ij}}^{ij} = \Lambda \quad \forall \text{ modes } ij$$

$$= \sum_{ij}\oint_{\mathbb{U}(\epsilon_{ij})} dx \,\ldots \text{etc.} \qquad\qquad \Omega(1+\epsilon_{ij}) = \Lambda$$

Then using the magnon's frequencies

$$\Omega(y) = \frac{1}{y^2-1}\left(1 - y\frac{X^+ + X^-}{1+X^+X^-}\right) \times \begin{cases} 1 & (i,j) \text{ light} \\ 2 & \text{heavy} \end{cases}$$

you would be led to

$$\epsilon_{\text{light}} = \frac{\sin^2\frac{p}{4}}{\Lambda} - \frac{\sin^2\frac{p}{2}}{8\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

$$\epsilon_{\text{heavy}} = 2\frac{\sin^2\frac{p}{4}}{\Lambda} + \frac{\sin^2\frac{p}{2}}{2\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) = 2\epsilon_{\text{light}} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

Only two.

And first terms enough precision — the integrals are log ϵ divergent.

Finite J Magnons

Corrections are organised like this:

$$E = \sum_{m,n=0,1,2\dots} a_{m,n} \left(e^{-\Delta/\sqrt{2\lambda}}\right)^m \left(e^{-2\Delta/E}\right)^n$$

- $a_{0,0} = E_{\text{class.}} + \delta E$ is the case $J = \infty$ from before.
- Corrections $a_{m,0}$ are ***F*-terms**, zero classically.
- Corrections $a_{0,n}$ are **μ-terms**, classical + one-loop, so we can make a comparison:

$$a_{0,1} e^{-2\Delta/E} = \sqrt{\frac{\lambda}{2}}\, a_{\text{class.}}(p,Q)\, e^{-2\Delta/E_0\left(\sqrt{\lambda/2},p,Q\right)} + \delta E^{\mu} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right) \quad \text{from algebraic c}$$

$$= h(\lambda)\, a_{\text{class.}}(p,Q)\, e^{-2\Delta/E_0(h,p,Q)} + a_{\text{subl.}} e^{-2\Delta/E_0} + \mathcal{O}\left(\frac{1}{h}\right) \quad \text{from Lüsher fo}$$

Will calculate δE^{μ} starting from classical finite-J solution.

Aside, **F-term corrections**.

For infinite-volume terms, we kept the first term in

$$\cot\left(\frac{q_i - q_j}{2}\right) = \pm i \left(1 + \underbrace{2e^{\mp i(q_i - q_j)}}_{\ell=1} + \underbrace{2e^{\mp 2i(q_i - q_j)}}_{\ell=2} + \dots\right).$$

For $a_{0,1}$ we need the second term:

$$\delta E^{F,1} = -\frac{1}{4\pi i}\sum_{\pm}\int_{\mathbb{U}_\pm} dx\, \partial_x \Omega_{ij}(x) \Big[\sum_{ij \text{ light}} (-1)^{F_{ij}} e^{\mp i(q_i - q_j)}\Big] \leftarrow \text{call this } F^{(\ell=1)}_{\text{light}}$$

$$= e^{-\Delta/\sqrt{2\lambda}} \sqrt{\frac{2\sqrt{2\lambda}}{\pi\,\Delta}} \left(\frac{\cos\frac{p}{2}}{1 - \sin\frac{p}{2}} - 1\right) \qquad \text{(saddle point } x = i\text{)}$$

... matching Lüscher calculation of [Bombardelli & Fioravanti, 2008].

For $a_{0,2}$ we need also the third term:

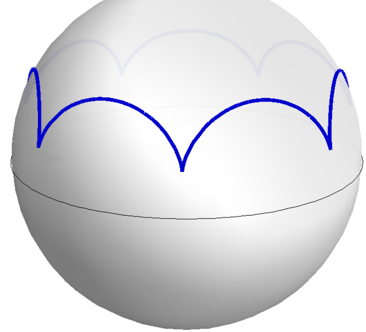
$$\delta E^{F,2} = -\frac{1}{2\pi i}\int_{\mathbb{U}_+} dx \left[\frac{1}{2} F^{(\ell=2)}_{\text{light}}\, \Omega'_{45}(x) + F^{(\ell=1)}_{\text{heavy}}\, 2\Omega'_{45}(x)\right]$$

$$= e^{-2\Delta/\sqrt{2\lambda}}\, 2\sqrt{\frac{\sqrt{2\lambda}}{\Delta\,\pi}} \left(\frac{\cos\frac{p}{2} - 1}{\sin\frac{p}{2} - 1}\right)$$

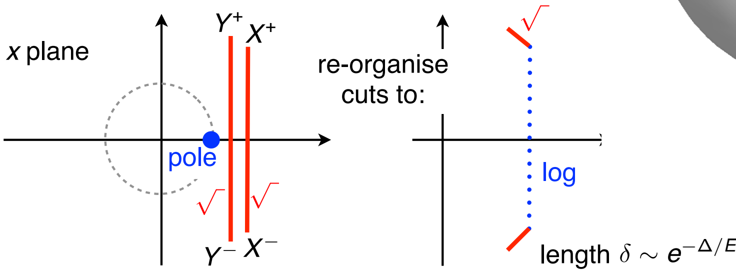
Heavy modes contribute starting at F,2. [MCA, Aniceto, Bombardelli, 2010]

No cutoff issues — terms are separately finite.

Finite-*J* classical solution



This is in fact the more natural case in the curve, where the magnon is a two-cut solution:



We expand everything in $\delta = |X^+ - Y^+|$, to order δ^2.
Elementary dyonic magnon (i.e. CP^2):

$$\delta E = -32 g^2 \cos(2\phi) \frac{Q}{E\sqrt{S(\frac{p}{2})}} \sin^3\left(\frac{p}{2}\right) e^{-\Delta E / S(\frac{p}{2})} + \ldots$$

where $S(\frac{p}{4}) = \frac{Q^2}{16 \sin^2(\frac{p}{4})} + 2\lambda \sin^2\left(\frac{p}{4}\right)$. [MCA, Aniceto, Ohlsson Sax, 2009]

CP^1 and RP^2, RP^3 cases just like S^5. [AFZ], [O&S] 2006, [H&S] [M& O-S] 2008

Efficient construction of off-shell frequencies $\Omega_{ij}(y)$.
Only $(1,5)$ and $(4,5)$ by hand, then: [Gromov, Schäfer-Nameki, Vieira, 2008]
[Bandres & Lipstein, 2009] for CP^3

- Inversion condition $\delta q_3(\frac{1}{y}) = -\delta q_4(y)$ gives

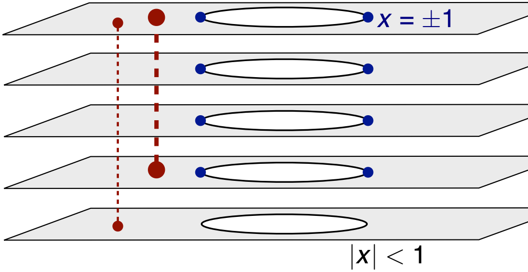
$$\Omega_{35}(y) = \Omega_{45}(0) - \Omega_{45}(\tfrac{1}{y}) \qquad \text{etc.}$$

  (using unphysical poles $|1/y| < 1$)

- Heavy modes are those without a pole on sheets 5, 6.
  These are constructed

$$\Omega_{17}(y) = \Omega_{15}(y) + \Omega_{45}(y) \qquad \text{etc.}$$

  (by canceling poles on sheets 5 or 6)



But on-shell, (i.e. evaluated at $y = x_n^{ij}$)
heavy modes are not superpositions.

Results:

Mode	(i,j)	Frequency $\Omega_{ij}(y)$
Light Fermions	$(1,5)$	$\Omega_{15}(y)$
	$(2,5)$	$\Omega_{15}(y)$
	$(1,6)$	$\Omega_{15}(y)$
	$(2,6)$	$\Omega_{15}(y)$
Light Bosons	$(3,5)$	$\Omega_{45}(0)-\Omega_{45}(\frac{1}{y})$
	$(3,6)$	$\Omega_{45}(0)-\Omega_{45}(\frac{1}{y})$
	$(4,5)$	$\Omega_{45}(y)$
	$(4,6)$	$\Omega_{45}(y)$
Heavy Fermions	$(1,7)$	$\Omega_{15}(y)+\Omega_{45}(y)$
	$(2,7)$	$\Omega_{15}(y)+\Omega_{45}(y)$
	$(1,8)$	$\Omega_{15}(y)+\Omega_{45}(0)-\Omega_{45}(\frac{1}{y})$
	$(2,8)$	$\Omega_{15}(y)+\Omega_{45}(0)-\Omega_{45}(\frac{1}{y})$
Heavy Bosons	$(1,9)$	$2\Omega_{15}(y)$
	$(1,10)$	$2\Omega_{15}(y)$
	$(2,9)$	$2\Omega_{15}(y)$
	$(3,7)$	$\Omega_{45}(y)+\Omega_{45}(0)-\Omega_{45}(\frac{1}{y})$

$$\Omega_{15}(y) = \frac{1}{y^2-1}\left(1 + y\frac{f(1)-f(-1)}{f(1)+f(-1)}\right)$$

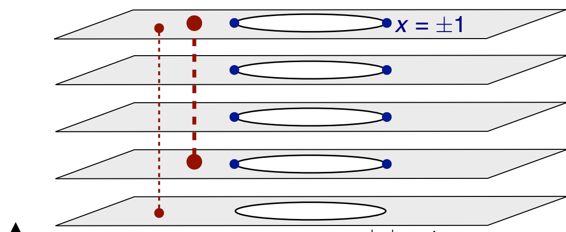
$$\Omega_{45}(y) = \frac{2}{f(1)+f(-1)}\left(\frac{f(y)}{y^2-1} - 1\right)$$

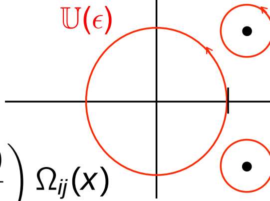
$$f(x) = \sqrt{(x-X^+)(x-Y^+)(x-X^-)(x-Y^-)}$$

$$\Omega_{ij}(y) = \Omega_{ij}^{(0)}(y) + \Omega_{ij}^{(2)}(y) + o(\delta^4)$$

$$\Omega_{15}^{(2)}(y) = \delta^2 e^{i2\phi} X_0^{+2} \frac{y}{y^2-1} \frac{X_0^+}{4(X_0^+X_0^-+1)^2} \frac{X_0^{-2}-1}{X_0^{+2}-1} -$$

$$\Omega_{45}^{(2)}(y) = \Omega_{15}^{(2)} + \left[\delta^2 e^{i2\phi} X_0^{+2} \frac{1}{y-X_0^+} \frac{X_0^+-X_0^-}{8(X_0^+X_0^-+1)(X_0^+\right.$$





Same integral as before:

$$\delta E^{\mathrm{INT}} = \frac{1}{4i} \sum_{ij} \oint_{-\mathbb{U}(\epsilon_{ij})} dx \, (-1)^{F_{ij}} \frac{q_i'(x) - q_j'(x)}{2\pi} \cot\left(\frac{q_i(x) - q_j(x)}{2}\right) \Omega_{ij}(x)$$

(plus some other terms) S^5 case: [Gromov, Schäfer-Nameki, Vieira, 2008]

What's new is worrying about cutoffs:

- **New** (same ϵ_{ij} $\forall ij$) works fine. (but not using $\sum^N + \sum^{2N}$!)
- **Old** (meaning $\epsilon_{\mathrm{heavy}} = 2\epsilon_{\mathrm{light}}$) leads to linear divergences...
- **Physical** works fine!
  Need precision $\epsilon = \frac{\#}{\Lambda} + \frac{\#}{\Lambda^2} + \frac{\#}{\Lambda^3}$ since $\sum_n^N \omega_n \sim N^2 \sim 1/\epsilon^2$ for each (i, j)

Simplest case: RP^3 magnons, which have same cut structure as S^5.

'New' matches [Bombardelli & Fioravanti, 2008]'s Lüscher calculation
with $c = 0$, and 'Physical' matches with $c = -\frac{\log 2}{2\pi}$.

[MCA, Aniceto, Bombardelli, 2011 i.p.]

For elementary magnon, both AC and Lüscher calculations are new...

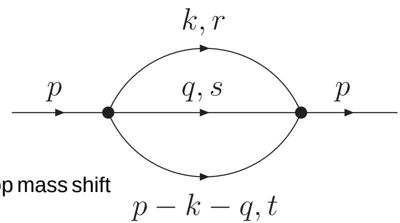
Near-Flat-Space Mass Corrections

Now no solitons, just strings near to “BMN vacuum” (pointlike $J \sim \sqrt{\lambda} \gg 1$ string) in a simplifying limit.

First the AdS_5 case, done by [Klose, McLoughlin, Minahan, Zarembo, 2007]. Dispersion relation, when $p_{\text{chain}} \sim \lambda^{-1/4}$:

$$E = \sqrt{1 + \frac{\lambda}{4\pi} \sin^2 \frac{p_{\text{chain}}}{2}}$$

$$E^2 = 1 + p^2 + \left[-\frac{\pi^2 p_-^4}{3m^2\lambda} \right]_{\text{two-loop mass shift}}$$



+ ...

The theory is this:

$$\mathcal{L} = \frac{1}{2}(\partial Y)^2 - \frac{m^2}{2} Y^2 + \frac{1}{2}(\partial Z)^2 - \frac{m^2}{2} Z^2 + \frac{i}{2} \psi \frac{\partial^2 + m^2}{\partial_-} \psi$$
$$+\gamma \, (Y^2 - Z^2)\big((\partial_- Y)^2 + (\partial_- Z)^2\big) + i\gamma \, (Y^2 - Z^2)\psi \partial_- \psi$$
$$+ i\gamma \, \psi \big(\partial_- Y^{i'} \Gamma^{i'} + \partial_- Z^i \Gamma^i\big)\big(Y^{i'} \Gamma^{i'} - Z^i \Gamma^i\big)\psi$$
$$-\frac{\gamma}{24}\big(\psi \Gamma^{i'j'} \psi \, \psi \Gamma^{i'j'} \psi - \psi \Gamma^{ij} \psi \, \psi \Gamma^{ij} \psi\big).$$

For δm^2:
only one integral,
and this is finite!

Repeat this in ABJM?
Steps:

1. Derive $\mathcal{L}^{\text{BMN}}$ as in [Sundin, 2010].
2. Take a large boost ($p_-/p_+ \sim \lambda^{1/2}$) and truncate.
3. Draw diagrams, and integrate.

We start from coset model

$$\frac{OSP(2,2|6)}{SO(1,3)\times U(3)}, \qquad A_\mu(\sigma,\tau) = -G^{-1}\,\partial_\mu G, \qquad A_\mu^{(0)} + A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)}$$

with action

$$S = \frac{g}{2}\int d^2\sigma\,\text{Str}\left[\sqrt{-h}h^{\mu\nu}A_\mu^{(2)}A_\nu^{(2)} + \epsilon^{\mu\nu}A_\mu^{(1)}A_\nu^{(3)}\right], \quad g = \sqrt{\frac{\lambda}{2}}$$

Fix light-cone gauge: $x^+ = \tau$ and $p_+ =$ const., breaks down to $SU(2|2)\times U(1)$.

Parameterisation of group element (adapted for gauge):

$$G = \underbrace{\Lambda(t,\phi)}_{\text{light-cone}} \cdot F(\chi) \cdot G_{AdS}(z_i) \oplus G_{CP}(y,\omega_\alpha).$$

with fields (light charged under the $U(1)$)

$\psi^a_\pm, \overline{\psi}_{\pm a}$	light fermion	$a = 1,2$	$SU(2)_L$	
$(s_\pm)^a_\alpha$	heavy fermion	$\alpha = 3,4$	$SU(2)_R$	
$\omega_\alpha, \overline{\omega}^\alpha$	light boson			$m = \frac{1}{2}$
$y,\ z_i$	heavy boson	$i = 1,2,3$		$m = 1$

Take BMN limit by scaling all fields $x \to \frac{1}{\sqrt{g}}x$, to get

$$S = \int d^2\sigma \left[\mathcal{L}_2(\pi,x,\chi) + \frac{1}{\sqrt{g}}\mathcal{L}_3(\pi,x,\chi) + \frac{1}{g}\mathcal{L}_4(\pi,x,\chi) + \mathcal{O}(g^{-3/2})\right]$$

as in [Sundin, 2010].

Free part is Lorentz invariant:

$$\mathcal{L}_2 = \frac{1}{2}\partial_+ y \partial_- y + \frac{1}{2}\partial_+ z_i \partial_- z_i + \frac{1}{4}\partial_+ \omega_\alpha \partial_- \overline{\omega}^\alpha + \frac{1}{4}\partial_- \omega_\alpha \partial_+ \overline{\omega}^\alpha - \frac{1}{2}\left(y^2 + z_i^2\right) - \frac{1}{8}\omega_\alpha \overline{\omega}^\alpha$$
$$+ i\left(\overline{\psi}_{+a}\partial_- \psi_+^a + \overline{\psi}_{-a}\partial_+ \psi_-^a\right) + \frac{i}{2}\left[(s_-)^a_\alpha \partial_+ (s_-)^\alpha_a + (s_+)^a_\alpha \partial_- (s_+)^\alpha_a\right]$$
$$- \frac{1}{2}\left(\overline{\psi}_{-a}\psi_+^a + \overline{\psi}_{+a}\psi_-^a\right) - i(s_+)^a_\alpha (s_-)^\alpha_a .$$

... but interaction terms are not.

Boost to near-flat-space:

(recall $g = \sqrt{\lambda/2}$)

$$\partial_\pm \to g^{\mp 1/2}\partial_\pm \qquad \text{(i.e. } \sigma^\pm \to g^{\pm 1/2}\sigma^\pm\text{)}$$
$$\psi_\pm \to g^{\mp 1/4}\psi_\pm \qquad \text{and likewise } s_\pm \to g^{\mp 1/4} s_\pm .$$

Leading terms in $\mathcal{L}_3$ go like $\sqrt{g}$, and we truncate to only such terms.
Similarly truncate $\mathcal{L}_4$ to terms $\sim g$.

Quantum consistency of this truncation is not obvious,
will contributions from small internal momenta cancel?

But in $AdS_5 \times S^5$, fine at two loops: [Klose, McLoughlin, Minahan, Zarembo, 2007]

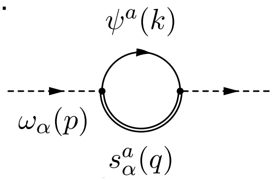
Cubic interaction

$$\mathcal{L}_3^{\rm NFS} = \frac{i}{8} y\,\omega_\alpha \overleftrightarrow{\partial_-}\, \overline{\omega}^\alpha + \frac{i}{2}\overline{\psi}_{-a} \overleftrightarrow{\partial_+}\, \psi^b_-\, \partial_- Z^a_b - \frac{1}{2}(\overline{\psi}_{+a}\partial_-\psi^b_- + \partial_-\overline{\psi}_{-a}\psi^b_+) Z^a_b$$
$$- \epsilon^{ab}\Big[\Big(\frac{3i}{16}(s_-)^\alpha_a\,\overline{\psi}_{-b} + \frac{i}{2}(s_+)^\alpha_a\,\partial_-\overline{\psi}_{-b} - \frac{1}{4}\partial_-(s_-)^\alpha_a\,\overline{\psi}_{+b} - \frac{i}{4}\partial_-(s_-)^\alpha_a\,\partial_+\overline{\psi}_{-b}$$
$$+ \frac{i}{4}\partial_+(s_-)^\alpha_a\,\partial_-\overline{\psi}_{-b}\Big)\omega_\alpha - \frac{i}{8}(s_-)^\alpha_a \overleftrightarrow{\partial_-}\, \overline{\psi}_{-b}\partial_+\omega_\alpha - \frac{i}{8}(s_-)^\alpha_a \overleftrightarrow{\partial_+}\, \overline{\psi}_{-b}\partial_-\omega_\alpha\Big]$$
$$+ \epsilon_{ab}\Big[\Big(\frac{3i}{16}(s_-)^a_\alpha\,\psi^b_- + \frac{i}{2}(s_+)^a_\alpha\,\partial_-\psi^b_- + \frac{1}{4}\partial_-(s_-)^a_\alpha\,\psi^b_+ - \frac{i}{4}\partial_-(s_-)^a_\alpha\,\partial_+\psi^b_-$$
$$+ \frac{i}{4}\partial_+(s_-)^a_\alpha\,\partial_-\psi^b_-\Big)\overline{\omega}^\alpha - \frac{i}{8}(s_-)^a_\alpha \overleftrightarrow{\partial_-}\, \psi^b_-\partial_+\overline{\omega}^\alpha - \frac{i}{8}(s_-)^a_\alpha \overleftrightarrow{\partial_+}\, \psi^b_-\partial_-\overline{\omega}^\alpha\Big].$$

Important points:

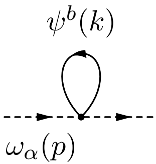
- It exists!
- All terms two light + one heavy
- Decorations ∂_+ occur (as do fields ψ_+) $\Leftarrow$ scalings $g^{\mp 1/4}\psi_\pm$, $g^{\mp 1/2}\partial_\pm$...

Consider terms $\frac{i}{4}\epsilon^{ab}\partial_+(s_-)^\alpha_a\,\partial_-\overline{\psi}_{-b}\omega_\alpha + \frac{i}{4}\epsilon_{ab}\partial_+(s_-)^a_\alpha\,\partial_-\psi^b_-\overline{\omega}^\alpha$.
These contribute to:



$$\approx \int d^2k\, \frac{k_-}{k^2 - \frac{1}{4}}\, \frac{q_-}{q^2 - 1}\, k_+^2 q_+^2$$
$$\approx p_-^2 \int_0^1 dx \int_0^\Lambda d\ell\,\ell\, \frac{\ell^4}{[\,\ell^2 - \Delta(x)\,]^2}$$
$$\sim \Lambda^2$$

Cancels with



$\sim \Lambda^2$.

[MCA & Sundin, 2011]

Results:

$$E = \sqrt{\frac{1}{4} + 4\,h(\lambda)^2 \sin^2 \frac{p_{\text{chain}}}{2}}, \qquad p_{\text{chain}} \sim \lambda^{-1/4}$$

can be expanded:

$$p_0^2 - p_1^2 = \frac{1}{4} + \Big[\frac{c\,p_-^2}{\sqrt{2\lambda}} - \underbrace{\frac{p_-^4}{96\lambda}}_{\text{two loops}}\Big] + \mathcal{O}\Big(\frac{1}{\sqrt{\lambda}}\Big) \qquad p_1 = \sqrt{\frac{\lambda}{2}}p_{\text{chain}}.$$

From diagrams,

$$\delta m^2 = -\frac{\log 2}{2\pi}\frac{p_-^2}{\sqrt{2\lambda}} \qquad \text{as expected,}$$

$$-\frac{3}{16\pi}\sqrt{\frac{2}{\lambda}}\,p_-^2 \qquad \text{unwanted!}$$

(The same using either dimensional regularisation or momentum cutoff Λ.)

Other particles:

Light fermion: $\psi_\pm$ $\quad \delta m^2 = -\frac{\log 2}{2\pi}\frac{p_-^2}{\sqrt{2\lambda}}$ (Perfect! But breaks supers

Heavy boson y $\quad \delta m^2 = -\frac{\log 2}{2\pi}\frac{p_-^2}{\sqrt{2\lambda}} - \frac{1}{8\pi}\sqrt{\frac{2}{\lambda}}\,p_-^2$

Heavy boson z $\quad \delta m^2 = -\frac{\log 2}{2\pi}\frac{p_-^2}{\sqrt{2\lambda}} + \frac{1}{4\pi}\sqrt{\frac{2}{\lambda}}\,p_-^2$

It this a **problem with near-flat-space** truncation? No.

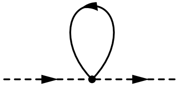
We repeated these calculations directly in BMN limit,
finding results which always reduce to near-flat ones:

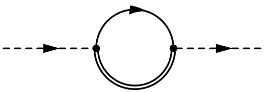
$$\delta m^2/\gamma = \underbrace{-\frac{5}{96\pi}}_{\text{invisible}} - \frac{3}{4\pi}p_1^2 \underbrace{- \frac{\log 2}{\pi}p_1^2}_{\text{expected}} \qquad \text{for light boson}$$

$$= -\frac{1}{96\pi} - \frac{\log 2}{\pi}p_1^2 \qquad \text{for light fermion.}$$

Fine-tuned cutoff? Try using $\begin{cases} \Lambda - \frac{\iota}{2\Lambda} & \text{light modes} \\ \Lambda - \frac{\varkappa}{2\Lambda} & \text{heavy modes} \end{cases}$

This has effect

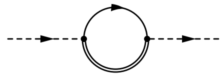
$\Lambda^2 \to \Lambda^2 - \iota$ for  (obviously!)

$\Lambda^2 \to \Lambda^2 - \dfrac{\iota + \varkappa}{2}$ for  (via Pauli-Villars)

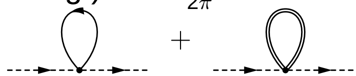
If you use $\iota = \frac{1}{4}$, $\varkappa = 1$ (which is $K^2 < \Lambda^2 - m^2 \; \forall$ masses)
this cancels half the unwanted term... [MCA & Sundin, 2011]

Apart from that...

- Note that we cannot implement the 'new sum', $\Lambda_{\text{heavy}} = 2\Lambda_{\text{light}}$, for the bubble term



However (strictly in dim. reg.) the $\frac{-\log 2}{2\pi}$ comes from tadpoles



Changing Λ for heavy tadpole removes this term... thus almost $c = 0$.

- The decay heavy$\rightarrow$light + light is allowed, in terms of energy:

$$\Delta m_y = -0.13\,\gamma p_-^2 \quad \rightarrow \quad 2\,\Delta m_\omega = -0.23\,\gamma p_-^2$$
$$\Delta m_z = -0.071\,\gamma p_-^2 \quad \rightarrow \quad 2\,\Delta m_\psi = -0.11\,\gamma p_-^2$$

Here $\Delta m = \frac{1}{2m}\delta m^2$ includes unwanted terms.

Without these, still allowed — we have $\delta m^2 = -c\,\frac{p_-^2}{\sqrt{2\lambda}}$ for both.

Conclusions

Heavy modes...

- Are simply 4 of the 8 $\perp$ directions in target space (+ fermions)
- Yet don't appear in the spin chain / Bethe equations.

There was no such issue in $AdS_5 \times S^5$.

- Are in some senses composite:
  . in off-shell construction of modes in algebraic curve,
  . (and as bound states in Lüscher calculations)
- ... and unstable:
  . compare shifted masses in near-flat-space.
- But must still be included in semiclassical calculations!

There are essentially two ways to cut these off:

“new” means *exactly* the same place in spectral plane: $|x| > 1 + \epsilon$,

gives $c = 0$,

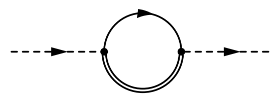
(approx. = $\sum^N$ light + $\sum^{2N}$ heavy, and energies Λ, 2Λ).

“physical” meaning *exactly* the same energy Λ,

gives $c = -\frac{\log 2}{2\pi}$,

(approx. = $\sum^N$ all, and also $\int_{1+\epsilon}$ all).

For near-flat-space calculation,



- No clear way to implement “new”
- And only imperfect implementation of “physical”?

The End.